RF–DC Differences of Coaxial Thermal Standards

DE-XIANG HUANG, MING-LU CHEN, AND SHUI-ZHI HE

Abstract—**The RF–dc differences (δ) of thermal converters are calculated from complex transmission line theory. The Boella effect and the skin effect are considered. The formula $\delta = Af^{1/2} + Bf^2$ can be obtained by experiments and proven by comparison of converters. The uncertainty of δ is 0.01 percent at 1 MHz and 0.1 percent at 30 MHz.**

I. Introduction

WITHIN THE RANGE of several tens of kilohertz to several tens of megahertz, the voltage standard is realized by means of RF coaxial thermal transfer standards and the accuracy may reach 0.1 to 0.01 percent. It cannot be determined with this accuracy by other standards. Therefore, how it can be estimated theoretically and verified by experiment becomes a problem of deep concern to metrologists worldwide.

II. The Influence of Distributed Parameters

In order to reduce error sources, the configuration of the RF coaxial thermal converter should be simple, as is shown in Fig. 1 [1]–[4]. It consists of a high-frequency rod resistor and an ultra-high-frequency thermoelement connected in series and mounted on the axis inside a metal cylinder. The input resistance is about 200 Ω/V at the maximum voltage of each range. The length of the low-range resistor is 12 mm or 51 mm and that of the high-range resistor is 51 mm. The input connector is a precision L16G straight type of RF socket, equivalent to the type "N" but manufactured in China to metric dimensions. In order to make experimental comparisons and estimate the influence of each parameter, different resistor films, resistor dimensions, thermoelements, cylinder dimensions, shield surface qualities, etc., are used. The reference plane for the voltage is the plane at the end of the external conductors. In making very accurate measurements, connections to other measurements are made directly or through precision tee adapters to reduce the influence caused by skin and transmission line effects. The RF–dc difference is generally defined by the equation

$$\delta = \frac{2V_{\mathrm{RF}}}{\left|V_{\mathrm{dc}}^{+}\right| + \left|V_{\mathrm{dc}}^{-}\right|} - 1 \tag{1}$$

where V_{RF}, V_{dc}^{+}, and V_{dc}^{-} are the radio frequency and direct (positive and negative) voltages required to produce the same output from the converter.

Manuscript received August 9, 1989; revised September 19, 1989.

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IEEE Log Number 8933402.

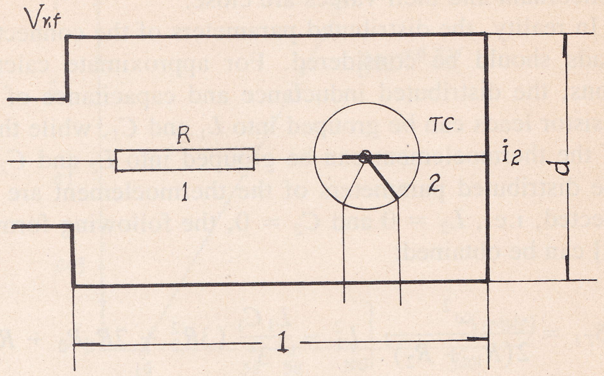

Fig. 1. The configuration of an RF converter.

A. The Effect of Transmission Lines

The RF–dc differences of coaxial thermal converters can be calculated from complex transmission line theory. Let $Z = R + j\omega L$ and $Y = j\omega C$, where R and L are the distributed parameters of the resistors or heating element and C is the capacitance to the shield. The voltage V_{RF} of the input plane can be obtained by the following:

$$V_{\mathrm{RF}} = \dot{I}_2 \sqrt{\frac{Z_2}{Y_2}} \sinh \sqrt{Z_2 Y_2} \cosh \sqrt{Z_1 Y_1} + \dot{I}_2 \sqrt{\frac{Z_1}{Y_1}} \sinh \sqrt{Z_1 Y_1} \cosh \sqrt{Z_2 Y_2} \tag{2}$$

where the subscript 1 refers to the parameters of the resistor, and the subscript 2 refers to those of the thermocouple. $\dot{I}_2$ is defined as the phasor current at the end of the heater. When each $\omega CR < 1$ and each $\omega L/R < 1$, by applying a Taylor's series expansion and neglecting higher order terms, we can derive the RF–dc differences δ_{T1} caused by transmission line effects [5]:

$$\begin{aligned}\delta_{T1} = \frac{\omega^2}{2(R_1 + R_2)^2} \Bigg[& (L_1 + L_2)^2 - \frac{L_1 C_1}{3} \cdot (R_1^2 + 2R_1R_2 + 3R_2^2) - \frac{L_2 C_2}{3} \\ & \cdot (R_2^2 + 2R_1R_2 + 3R_1^2) \\ & - \frac{2}{3} C_1 L_2 R_1^2 - \frac{2}{3} C_2 L_1 R_2^2 + R_1^2 C_1^2 \\ & \cdot \left(\frac{R_1^2}{90} + \frac{R_1 R_2}{15} + \frac{R_2^2}{6}\right) \\ & + R_2^2 C_2^2 \left(\frac{R_2^2}{90} + \frac{R_1 R_2}{15} + \frac{R_1^2}{6}\right) + \frac{2C_1 C_2 R_1^2 R_2^2}{9}\Bigg].\end{aligned} \tag{3}$$

A more accurate solution can be obtained if we use the corresponding Maxwell equations, but it is quite troublesome. We try to use (3) as a first approximation for two kinds of resistors and the heaters. We compare the calculated results with the experimental data; their trends are concordant and their values are close.

In reality, the distributed parameters of the connecting leads should be considered. For approximate calculations, the distributed inductance and capacitance of the resistor leads can be grouped into L_1 and C_1, while those of the thermoelement can be grouped into L_2 and C_2. If the distributed parameters of the thermoelement are neglected, i.e., $L_2 = 0$ and $C_2 = 0$, the following formula [1] can be obtained:

$$\delta_{T1} = \frac{\omega^2}{2(R_1 + R_2)^2}\left[L_1^2 - \frac{L_1 C_1}{3}(3R_2^2 + 2R_1R_2 + R_1^2) + C_1^2 R_1^2\left(\frac{R_1^2}{90} + \frac{R_1R_2}{15} + \frac{R_2^2}{6}\right)\right]. \quad (4)$$

In fact, the phasor current at the middle of the heater, $\dot{I}_h$, is not equal to $\dot{I}_2$; this influence is small and can be calculated approximately from

$$\delta_{T2} = \frac{\omega^2}{8}\left(L_2C_2 - \frac{1}{24}C_2^2R_2^2\right). \quad (5)$$

For a typical thermoelement, δ_{T2} is equal to approximately +0.08 percent at 100 MHz.

Let δ caused by the input connector be designed as δ_{T3}, which can be calculated by conventional transmission line formulas.

B. *The Boella Effect*

The effects of distributed inductance of the resistors and the distributed capacitance to the shield have already been considered in (2). However, displacement current inevitably exists in the resistors, and also causes some dielectric loss. In wireless engineering, the equivalent circuit is usually expressed as that of a resistor connected to shunt the distributed capacitance between terminals, between various parts of the resistor and the terminals, and between parts of the resistor [6], [7]. This leads to the impedance dropping with increasing frequency. This is known as the Boella effect. The ratio of conductive current to displacement current is proportional to $4\pi\sigma/\epsilon\omega$, where σ is conductivity and ϵ is the dielectric constant. To obtain a larger value of $4\pi\sigma/\epsilon\omega$, the resistors should be made with small cross sections so that the Boella effect is reduced. Obviously thin-film resistors are better than composition resistors. Such a resistor may be regarded as a transmission line having uniformly distributed series resistance and capacitance. The distributed capacitance can be calculated according to its shape and dimensions. Its impedance is $Z = \sqrt{(R/j\omega C)}\tanh(1/2)\sqrt{R \cdot j\omega C}$ [8]. If only the Boella effect is considered, when $\omega CR < 0.6$, the simplified equivalent circuit of the converter is shown in Fig. 2, where C_r is the equivalent capacitance of the

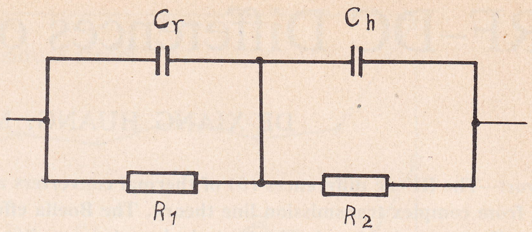

Fig. 2. The equivalent circuit of the Boella effect.

resistor, and C_h that of the heater. Then we obtain

$$\delta_B = \frac{\omega^2}{2(R_1 + R_2)^2}\left[2C_hC_rR_1^2R_2^2 - C_r^2R_1^2(R_1^2 + 2R_1R_2) - C_h^2R_2^2(R_2^2 + 2R_1R_2)\right]. \quad (6)$$

The calculation shows that C_h is much smaller than C_r; therefore

$$\delta_B \cong \frac{-\omega^2}{2(R_1 + R_2)^2} C_r^2R_1^2(R_1^2 + 2R_1R_2).$$

C. *δ_d Caused by Distributed Parameters*

This is given by

$$\delta_d = \delta_{T1} + \delta_{T2} + \delta_{T3} + \delta_B = Bf^2$$

where B is a coefficient whose theoretical values are listed in Table I.

These theoretical values can be used as guidance for the experiments. While they are close to the δ values of the actual converters, they are not used as the actual values of δ of the converters.

To minimize the δ_d of the 0.5- and 1-V ranges, the dimensions of converters must be reduced as much as possible.

III. The Influence of Skin Effect

The connecting leads of vacuum thermoelements are usually made of ferromagnetic materials in order to match the thermal expansion coefficient of its glass shell. In this case, the influence of the skin effect must be considered [9].

Surely the skin effect in both the internal and external leads would increase the ac resistance and reduce the heating current so that it has differing effects on voltage converters of various ranges. However, the internal leads will also increase the heating resistance. (See the Appendix.) Calculations show that the shorter the external leads of the 0.5- and 1-V converters the better. Other factors, such as the shield of the converter, must also be considered. The shield of the converter is made of brass. It is necessary to have smooth surfaces and a uniform coating of a high conductive material such as silver in order to reduce the effect of surface reactance.

The RF–dc differences δ_s caused by skin effect are proportional to the square root of their frequencies. Thus $\delta_s = Af^{1/2}$. The lower frequency limit for this is approximately 50 kHz for the copper-coated magnetic leads and 20 kHz for the platinum leads. Therefore, above these

The skin effect of heater is negligible if compare

TABLE I
THE THEORETICAL VALUE OF δ_d

f (MHz)	$\delta_{1V} - \delta_{2V}$ (%)	$\Delta\delta$ (%)	Difference (%)
1	0.0076	0.0084	-0.0008
10	0.036	0.0315	0.0045
30	0.157	0.168	-0.011

two frequencies, the RF–dc differences δ can be theoretically expressed as

$$\delta = \delta_s + \delta_d = Af^{1/2} + Bf^2. \tag{7}$$

IV. EXPERIMENTAL RESULTS

The audio-frequency error of an RF coaxial thermal converter is very small, and can be directly measured using RF thermal transfer standards. The errors of converters at 100 MHz and above are larger, and can be measured directly by means of thin-film bolometer bridges, similar to Selby's Bolovac device at NIST. There are two thermoresistors on a thin film which is mounted in the coaxial line and they form an arm of a bridge. The primary national standard of China is such a bridge and has an accuracy of 0.25 percent from 10 MHz to 1 GHz [10]. The most important problem is how to determine the errors at frequencies of 20 kHz–50 MHz.

A. Original Converters

The RF–dc differences of actual converters, especially those of compensated converters, are a complicated function of frequency. These differences are systematic errors and it does not matter much that these differences are great if they are stable and can be determined accurately. In order to reduce error sources, converters of the simple geometry shown in Fig. 1 were made. Even if the RF–dc differences of the original converter are rather pronounced, high accuracy can be obtained by applying corrections, as long as they follow a definite rule and their values can be determined precisely.

The RF–dc differences of the original converter ranging from 0.5 to 5 V as measured using the thin-film bolometer bridge are less than 17 percent for frequencies up to and including 300 MHz. The values of δ for actual converters of the same type and workmanship will generally deviate from their average values of up to ± 0.015 percent at 30 MHz and ± 0.15 percent at 100 MHz due to fabrication variations.

According to the experimental results mentioned above, a satisfactory approximate empirical formula can be derived from either graphical extrapolation or calculation. The expression is [11]

$$\delta = Kf^{\alpha} \tag{8}$$

where $\alpha = 2$ or slightly less. Referring to Fig. 3, the random measuring errors can be reduced by these methods. This formula can be proved by direct comparison of converters. However, when frequencies are below 10 MHz, formula (8) cannot be applied. The value of δ in (8) at 1 MHz is very small and that of $(\delta_{1\,V} - \delta_{2\,V})$ is also very small, but the $\Delta\delta$ derived from the direct comparison of 1- and 2-V converters is larger than the value of $(\delta_{1\,V} - \delta_{2\,V})$.

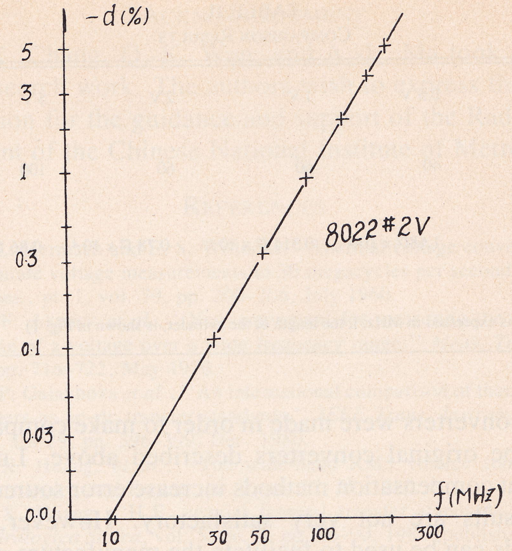

Fig. 3. Graph of $(f\text{–}\delta_d)$.

This is a problem. Several other formulas were tried without success. Finally, (7) was employed to process the experimental data. However, $\delta = Kf^{\alpha}$ has been found by extrapolation to reduce random error in the past. Thus one can substitute δ and f of (8) into (7) to obtain A and B by the method of least squares. The results of using (7) are very close to those resulting from using (8) when frequencies are higher than 10 MHz. The advantage of using (7) is that it very clearly exhibits the physical concept, and is applicable from 50 kHz up to 100 MHz.

Is it reliable to obtain δ from $\delta = Af^{1/2} + Bf^2$ (formula (7))? Direct comparison of two converters is the best verification method. The converters of 1 and 2 V generally have δ's of reversed signs, e.g.

$$8021\#1\ \mathrm{V}\ \delta = (50f^{1/2} + 0.487f^2) \times 10^{-4}$$

$$8022\#2\ \mathrm{V}\ \delta = (-25f^{1/2} - 0.829f^2) \times 10^{-4}$$

where f is in megahertz and δ is in percent. Notice that the A of 2 V is caused mainly by the internal leads of the thermoelements. The $\Delta\delta$ obtained from the direct comparison of 2 and 1 V is almost equal to the values of $(\delta_{1\,V} - \delta_{2\,V})$ derived from (7). The accuracy of δ is thus verified. Some of the experimental results are listed in Table II. Similar experiments on 0.5 and 1 V, as well as on 2- and 5-V converters yield similar agreement.

B. Compensated Converters

Compensation methods can be classed into two groups: configuration compensation and element compensation. There are many different methods for different voltage ranges and different frequency ranges. Many compen-

TABLE II
COMPARISON RESULTS

range	0.5 V	1 V	2 V	5 V
l (mm)	40	60	60	100
δ_d (%)	$1.68 f^2 \times 10^{-4}$	$0.314 f^2 \times 10^{-4}$	$-0.78 f^2 \times 10^{-4}$	$-0.86 f^2 \times 10^{-4}$

(f is expressed in MHz; l, the length of the cylinder, is shown in Fig. 1)

sated converters were made in order to make comparisons with the original converters described above. Lumped-element compensation methods increase error sources, and the results are not very satisfactory. However, these methods can be used to facilitate the manufacture of thermal voltage standards as products.

Based on the above-mentioned experiments, a new type of 101#1-V converter with external shield and dual internal leads has been designed. The influence of distributed parameters and skin effect has been reduced as much as possible by excellent workmanship and compensation. The converter 101#1 V was measured using the bolometer bridge (Chinese standard), and the measured value of δ of converter 101#1 V (not the uncertainty) is less than 0.1 percent up to 300 MHz. This can be considered a suitable voltage standard from 10 to 100 MHz. However, when we compared 101#1 V with 8021#1 V and 8022#2 V, errors were found below 100 MHz, the RF–dc differences of 101#1 V being 29 ppm (1 ppm = 0.0001 percent) at 1 MHz and 200 ppm at 30 MHz. This is a further verification of the results mentioned for the original converters.

Through theoretical and error analyses of the 101#1-V converter, it was found that the error is caused mainly by the skin effect, and we believe that the methods used in the analysis of the original converters appear to be more accurate.

Other configuration-compensated converters are also similar. Therefore, in our RF voltage standards system, 8022#2 V and other original converters serve as the primary standards, 101#1 V serves as the reference standard, and other compensated converters are the working standards. For higher voltage ranges, the accuracies can be obtained through comparisons.

C. Stability

Since it is mainly determined by its geometry and the materials used, the systematic error of an RF coaxial thermal converter has a long-sustained stability. For example, the change of the RF–dc differences of 8022#2 V over five years is approximately 18 ppm at 1 MHz and 150 ppm at 30 MHz. This has been determined from measurements using the thin-film bolometer bridge and through the complex process discussed above. Further analyses show that the change is mainly caused by the change of the surface reactances of the shield, leads, and connectors.

Thus the converter is very suitable for use as a precision voltage standard, and the RF–dc differences of the primary standard converters should be redetermined every three to five years.

D. Uncertainty

Precise values of δ can be obtained through processing the experimental results, as described above, or by comparing the converter with a standard converter. After obtaining the δ values, we can get

$$V_{\mathrm{RF}} = \left[\frac{|V_{\mathrm{dc}}^{+}| + |V_{\mathrm{dc}}^{-}|}{2} \right] (1 + \delta).$$

Thus the error of RF voltage is very much reduced. However, what about the systematic uncertainty of δ? It can be derived from differentiating (7) and (8), as well as from carrying out precise and elaborate calculations based on experimental data. The uniformity of extrapolation and direct comparison is the best proof of the experiment.

V. CONCLUSION

Leaving sufficient margin, the 3-σ uncertainties of the RF–dc differences are given at the bottom of Table III.

The method of treating experimental data also applies to micropotentiometer and other voltage standards.

APPENDIX
THE FORMULAS FOR δ VALUES CAUSED BY THE SKIN EFFECT

Let

$$R_{e\mathrm{RF}} - R_e = \Delta_e \qquad R_{i\mathrm{RF}} - R_i = \Delta_i.$$

Here $R_{e\mathrm{RF}}$ stands for the RF resistance of the external leads and cylinder, R_e the dc resistance, and $R_{i\mathrm{RF}}$ expresses the RF resistance of the internal leads, and R_i the dc resistance.

The RF–dc differences caused by the external leads of the thermoelement and cylinder is given by

$$\delta_{s_1} = \frac{R_{e\mathrm{RF}} - R_e}{R_1 + R_2} = \frac{\Delta_e}{R_1 + R_2}.$$

The RF–dc differences caused by the internal leads are determined from

$$\frac{V_{\mathrm{dc}}^2}{(R_1 + R_2 + R_i)^2}(R_2 + R_i) = \frac{V_{\mathrm{RF}}^2}{(R_1 + R_2 + R_{i\mathrm{RF}})^2}(R_2 + R_{i\mathrm{RF}})$$

$$\frac{V_{\mathrm{RF}}}{V_{\mathrm{dc}}} = \left(\frac{R_1 + R_2 + R_{i\mathrm{RF}}}{R_1 + R_2 + R_i}\right)\sqrt{\frac{R_2 + R_i}{R_2 + R_{i\mathrm{RF}}}} \cong 1 + \frac{(R_2 - R_1)\cdot\Delta_i}{2R_2(R_1 + R_2)}$$

$$\therefore\ \delta_{S_2} = \frac{V_{\mathrm{RF}}}{V_{\mathrm{dc}}} - 1 \cong \frac{R_2 - R_1}{2R_2(R_1 + R_2)}\cdot\Delta_i.$$

TABLE III
ERROR SOURCES AND ESTIMATES OF THEIR EFFECTS ON THE MEASUREMENT (IN PPM PARTS PER MILLION)

Error Source	≤ 1MHz	10MHz	30MHz	50MHz	100MHz
AC-DC difference at 1 kHz	10	10	10	10	10
Influence of distributed parameters	3	30	100	250	700
Skin effect	10	30	60	70	100
Error of Tee and connector	3	5	20	40	100
Comparator system uncertainty	10	20	50	100	200
Long-term stability (3 years)	15	40	100	200	500
RSS sum	23	63	163	345	894
A conservative estimate of uncertainty	100	300	1000	2000	3000

The RF–dc differences caused by the external and internal leads are

$$\delta_S = \delta_{S_1} + \delta_{S_2} = \frac{\Delta_e + \Delta_i}{R_1 + R_2} - \frac{\Delta_i}{2R_2}.$$

ACKNOWLEDGMENT

Y.-X. Chang, Q.-K. Wan, and A.-L. Ma took part in this research work. The authors wish to express their appreciation for the guidance and support of the Radio Department of the Chinese National Institute of Metrology.

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